Design and Analysis of LDPC for MIMO-OFDM

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Outline

- LDPC coded MIMO OFDM
- Analysis & Optimization of (irregular) LDPC Coded MIMO OFDM
  - A few practical issues: Different number of antennas; different MIMO demodulation schemes; different spatial correlation models
  - Large-code-length: Optimization of degree profiles by density evolution with Gaussian approximation
  - Short-code-length: Random construction with girth conditioning
- Numerical examples and conclusions

Problem Statement

- Future personal wireless communications
  - A popular vision: IP-based multimedia wireless services with both ubiquitous coverage ($\geq$ cellular) and high speed ($\geq$ Wi-Fi).
  - A narrow-sense engineering vision: wireless packet IP data communications with high throughput and low latency.
- Enabling techniques for high-speed wireless packet data
  - *PHY layer*: MIMO, advanced FEC, advanced DSP, adaptive transmission, ...
  - *MAC layer*: channel-aware scheduling, multi-access, fast ARQ, interference control, ...
  - *Networking layer, cross-layer, . . .*
- In this work, we focus on the peak date-rate of downlink transmission

Low-Density Parity-Check (LDPC) Codes

- Invented by R. Gallager in 1962; re-discovered by Mackay & Neal in 1997, by Richardson & Shokrollahi & Urbanke in 1999.
- LDPC is a linear block code defined by a very sparse parity check matrix; or equivalently by a bipartite (Tanner) graph (variable nodes, check nodes and connecting edges).
- LDPC codes subsume a class of capacity-approaching codes, e.g., turbo codes, RA codes.
- Decoding complexity of LDPC codes is lower than turbo codes, and suitable for parallel processing.

◇ *Regular LDPC codes*: same number of 1's in each column and row of the sparse parity check matrix.

◇ *Irregular LDPC codes*: different number of 1's ...... Large-code-size irregular LDPC: *degree profiles*.

▶ *Deterministic LDPC construction*: array codes [Fan '99], graph theory [Lin '02], . . .

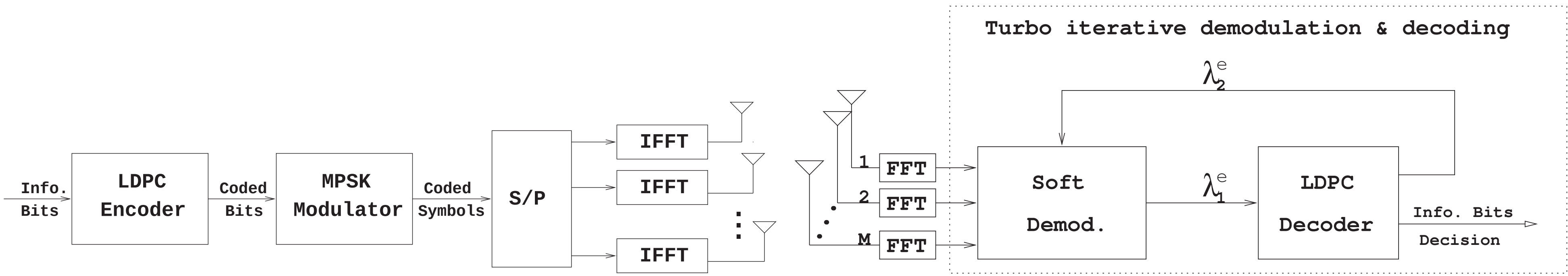
▶ *Pseudo-random LDPC construction*: convergence to ensemble average theorem for large-code-size [Gallager 63'], girth conditioning for moderate/short-code-size [Campeliot & Modha & Rajagopalan 99', Yang & Ryan 02', Tian & Jones & Villasenor & Wesel 02'].

LDPC Code Optimization

- Previous works on LDPC optimization
  - for AWGN channels by density evolution [Richardson & Shokrollahi & Urbanke, 01']
  - for AWGN channels by density evolution with Gaussian approx [Chung & Forney & Richardson & Urbanke, 01']
  - for Rayleigh fading channels by density evolution with mixture Gaussian approx [Hou & Siegel & Milstein, 01']
  - for ISI channels by density evolution with mixture Gaussian approx [Narayanan & Wang & Yue, 02']
  - for MIMO channels by EXIT Chart [tenBrink & Kramer & Ashikhmin, 02',]
  - ...
- In this work
  - optimization for MIMO OFDM channels by density evolution with mixture Gaussian approx.
    * *number of antennas and bandwidth*: use of MIMO technique to support the same data rate with less bandwidth (i.e., higher spectral efficiency).
    * *low-complexity iterative receiver*: use of low-complexity soft LMMSE-SIC MIMO demodulator, as opposed to exponentially complex soft MAP MIMO demodulator.
    * *spatially correlated MIMO*: non-full-scattering scenario (due to limited antenna separation or angle spread)

LDPC Coded MIMO OFDM for 4G Downlink

- *MIMO*: multiple-antennas at both transmit and receive sides; establish the multi-fold virtual air-links, the spatial resource not regulated by FCC.
- *OFDM*: low-complexity in dispersive channels; easy bond with multiuser scheduler; a highly competitive solution for (synchronous) downlink transmission.
- *LDPC*: capacity-approaching; low-complexity & parallizable decoder; freedom for design and performance optimization.



Turbo Iterative Demodulation and Decoding

[1] *Iteration of turbo receiver:* For $q = 1, 2, \ldots, Q$

[1-a] *Soft MIMO OFDM demodulation:* $L^{q}_{D\to L}[b_i] = g(\{r(t)\}, \{L^{q-1}_{D\leftarrow L}[b_j]\}_j)$,

[1-b] *Soft LDPC decoding:* For $p = 1, 2, \ldots, P$

Sum-product algorithm: for all variable nodes and check nodes

Variable node update: $L^{p,q}_{b\to c}(e^b_{i,j}) = L^q_{m\to L}[b_k(i)] + \sum_{n=1,n\neq j}^{\nu_i} L^{p-1,q}_{b\leftarrow c}(e^b_{i,n})$.

Check node update: $L^{p,q}_{b\leftarrow c}(e^c_{i,j}) = 2\tanh^{-1}\left[\prod_{n=1,n\neq j}^{\Delta_i}\tanh\left(\frac{L^{p,q}_{b\to c}(e^c_{i,n})}{2}\right)\right]$.

[1-c] *Compute extrinsic messages passed back to the multiuser detector:*

$$L^q_{D\leftarrow L}[b_i] = \sum_{n=1}^{\nu_i} L^{P,q}_{b\leftarrow c}(e^b_{i,n}).$$

[2] *Final hard decisions on information and parity bits:*

$$\hat{b}_i \;=\; \operatorname{sign}\left\{L^Q_{D\to L}[b_i] + L^Q_{D\leftarrow L}[b_i]\right\}.$$

Analysis & Optimization of LDPC Coded MIMO OFDM

- Degree profiles of LDPC: $\lambda(x) = \sum_{i=1}^{d_{l_{max}}} \lambda_i \; x^{i-1}$ and $\rho(x) = \sum_{i=1}^{d_{r_{max}}} \rho_i \; x^{i-1}$
- Optimization problem

$$(\lambda^*(x), \rho^*(x)) = \arg \min_{\lambda(x),\rho(x)} \text{SNR} : \left| \left\{ L^Q_{D\to L}[b_i] + L^Q_{D\leftarrow L}[b_i] \right\} \right| \to \infty.$$

- Basic idea: track the dynamics of turbo iterative demodulation and decoding.
- Major assumptions and approximations
  - Assume the extrinsic LLR at each variable node or check node of LDPC codes is Gaussian and symmetric, i.e., $\mathcal{N}(m, 2m)$.
  - Assume the LLR from LDPC decoder to MIMO demodulator as mixture Gaussian $f^q_{D\leftarrow L} \cong \sum_{j=2}^{d_{l,max}} \tilde{\lambda}_j \; \mathcal{N}(m_j, 2m_j)$. – due to sum-product algorithm
  - Approx the LLR from MIMO demodulator to LDPC decoder as mixture Gaussian $f^q_{D\to L} \cong \sum_{i=1}^{J} \pi_i \; \mathcal{N}(m_i, 2m_i)$. – using EM algorithm
- We then only need to track parameters of mixture Gaussian's, $\{\pi_i, m_i\}_i$, rather than complete pdf's.

Analysis & Optimization of LDPC MIMO OFDM

- *Turbo receiver iterations:* For $q = 1, 2, \ldots, Q$
  - *Mixture Gaussian approx of extrinsic LLR of MIMO demodulator:*

$$f^q_{D \to L} = \sum_{j=1}^{J} \pi_j \mathcal{N}(\mu_j, 2\mu_j)$$

  - *Mixture Gaussian approx of extrinsic LLR of LDPC decoder:*
    - □ *Iterate between variable node update and check node update:* For $p = 1, 2, \ldots, P$
      - ◇ *At a bit node of degree i:*

$$f^{p,q}_{b \to c} = \sum_{j=1}^{J} \sum_{i=2}^{d_{l,max}} \pi_j \lambda_i \mathcal{N}\Big(\mu_j + (i-1) m^{p-1,q}_{b \leftarrow c},\ 2[\mu_j + (i-1) m^{p-1,q}_{b \leftarrow c}]\Big)$$

      - ◇ *At check node of degree j:*

$$f^{p,q}_{b \leftarrow c} = \sum_{j=2}^{d_{r,max}} \rho_j \mathcal{N}\left(m^{p,q}_{b \leftarrow c,j},\ 2m^{p,q}_{b \leftarrow c,j}\right)$$

    - □ *Message passed back to the multiuser detector:*

$$f^{P,q}_{D \leftarrow L} = \sum_{i=2}^{d_{l,max}} \tilde{\lambda}_i \mathcal{N}\left(m^q_{D \leftarrow L}(i),\ 2m^q_{D \leftarrow L}(i)\right)$$

- The optimized SNR threshold

$$(\lambda^*(x), \rho^*(x)) = \arg \min_{\lambda(x), \rho(x)} \text{SNR} : \left| \left\{ L^Q_{D \to L}[b_i] + L^Q_{D \leftarrow L}[b_i] \right\} \right| \to \infty.$$

Performance in Ergodic Channels w/o Spatial Correlation

- Within 1.0 dB from channel capacity

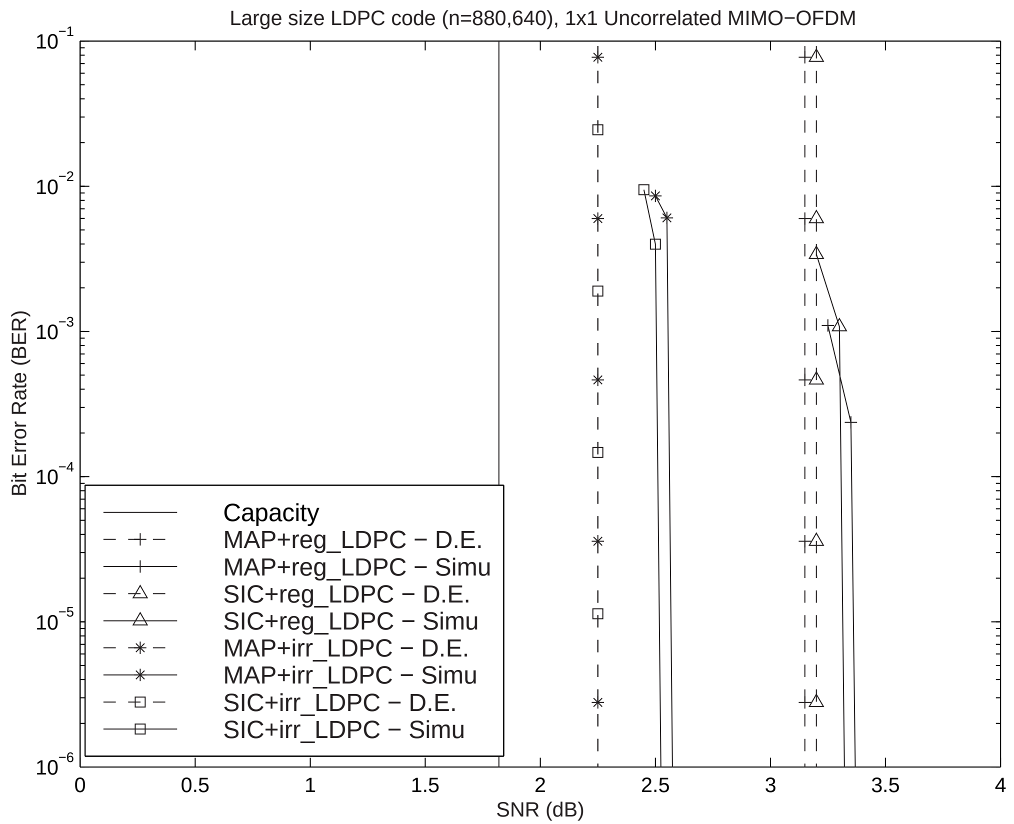


Figure 1: Large-block-size LDPC in 1×1 MIMO OFDM.

Performance in Ergodic Channels w/o Spatial Correlation

- Within 1.0 dB from channel capacity

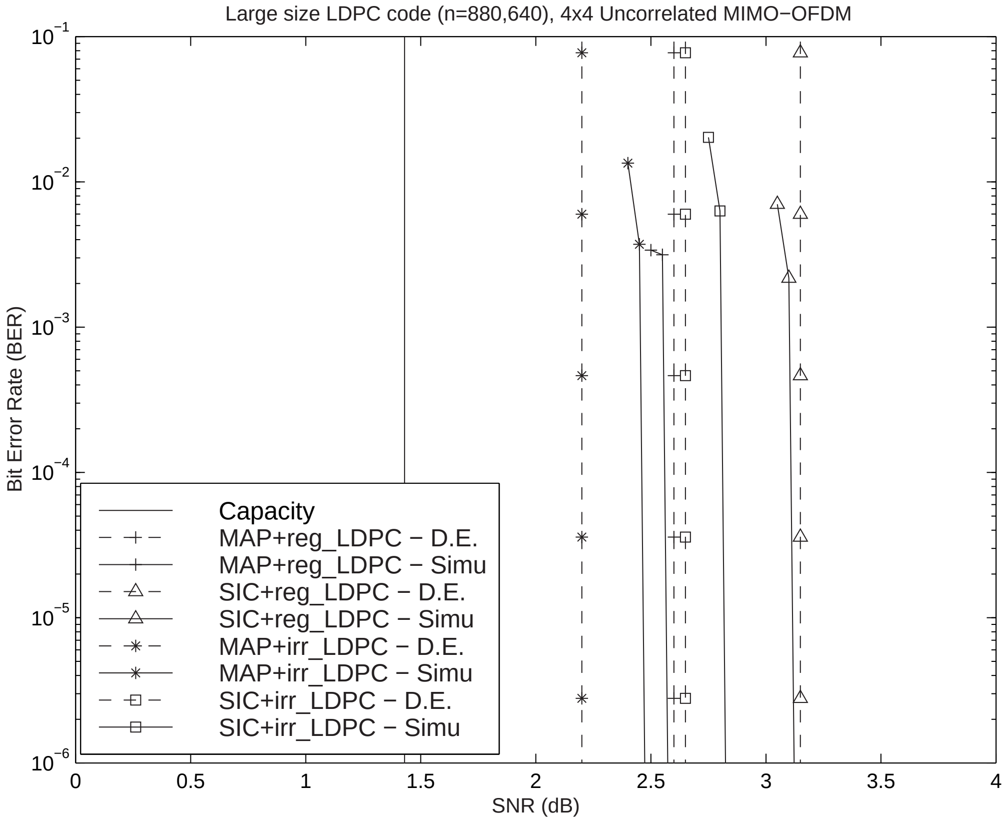


Figure 2: Large-block-size LDPC in 4×4 MIMO OFDM.

Performance in Ergodic Channels with Spatial Correlation

- LMMSE-SIC demodulator suffers extra loss due to spatial correlation

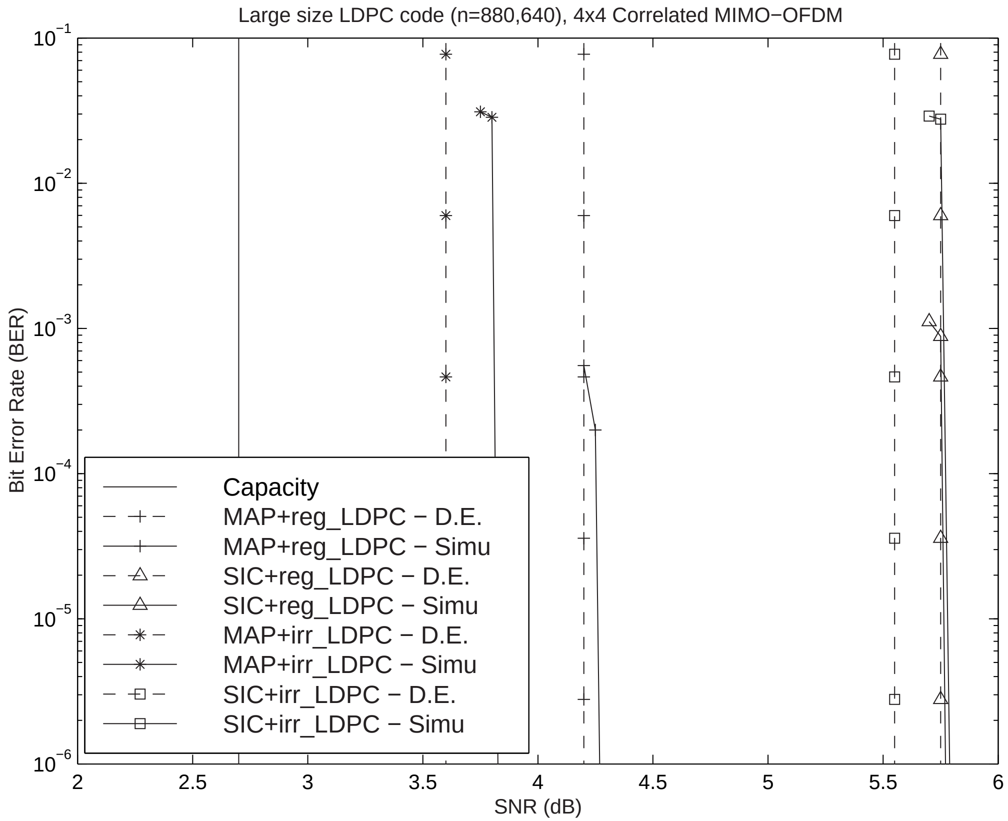


Figure 3: Large-block-size LDPC in 4×4 MIMO OFDM.

Performance in Outage Channels

- Within 1.5 dB from channel capacity

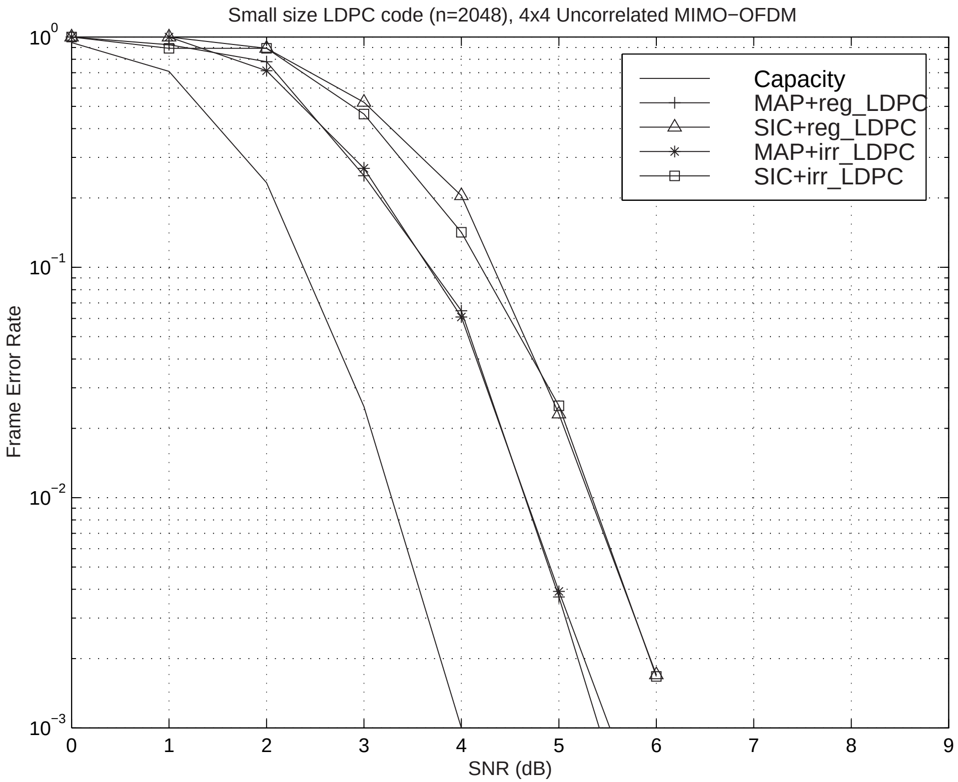


Figure 4: Short-block-size LDPC in 4 × 4 MIMO OFDM, target FER of 10^{-2}.

Performance in Outage Channels: Convergence of Turbo Iterative Receiver

- Irregular LDPC expedites the convergence of overall turbo receiver

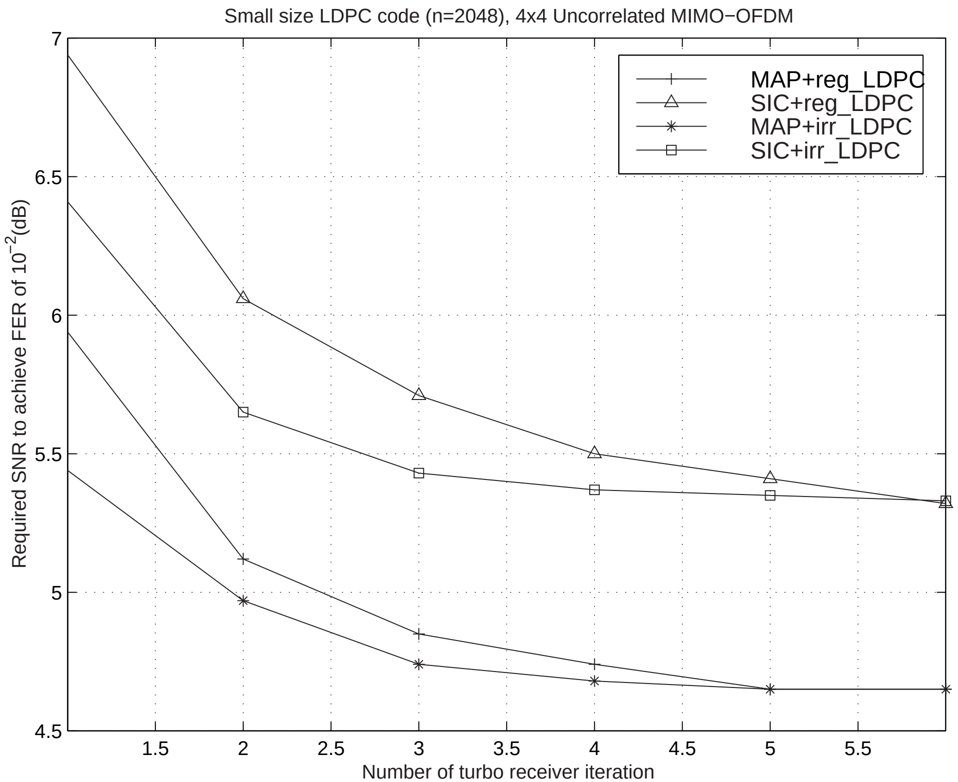


Figure 5: Short-block-size LDPC in 4×4 MIMO OFDM, target FER of 10^{-2}.

Gain of Channel-Specific LDPC Design

- Design gain of MIMO-OFDM-optimized LDPC increases for larger number of antennas, as compared to AWGN-optimized LDPC.

	Large Block Irregular LDPC			Small Block Irregular LDPC		
SNR (dB)	LDPC.I	LDPC.II	Channel-specific Design Gain (LDPC.II - LDPC.I)	LDPC.I	LDPC.II	Channel-specific Design Gain (LDPC.II - LDPC.I)
MAP (1×1)	2.57	2.57	0.00	7.08	7.08	0.00
MAP (2×2)	2.56	2.61	0.05	5.57	5.72	0.15
MAP (4×4)	2.46	2.65	0.19	4.48	4.81	0.33
SIC (1×1)	2.52	2.52	0.00	7.06	7.06	0.00
SIC (2×2)	2.75	2.92	0.17	6.32	6.44	0.12
SIC (4×4)	2.82	3.17	0.35	5.33	5.70	0.37

LDPC.I: Performance of MIMO-OFDM-optimized LDPC in MIMO-OFDM channels.

LDPC.II: Performance of AWGN-optimized LDPC in MIMO-OFDM channels.

Summary

- LDPC coded MIMO OFDM is capable of supporting 4G wireless packet data transmission with higher spectral efficiency – which translates into either bandwidth saving or further data rate increase.
- In ergodic channels, channel-specific (irregular) LDPC optimization results in larger SNR gain in systems with larger number of antennas.
- In outage channels, irregular LDPC codes lead to faster receiver convergence.
- LMMSE-SIC based receiver performs near-optimal in spatially uncorrelated MIMO OFDM channels; but suffers additional loss in MIMO channels with severe spatial correlation.